Horizontal Curves- Superelevation- Vertical Curves

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4th year Civil

weeks 10- 11- 12

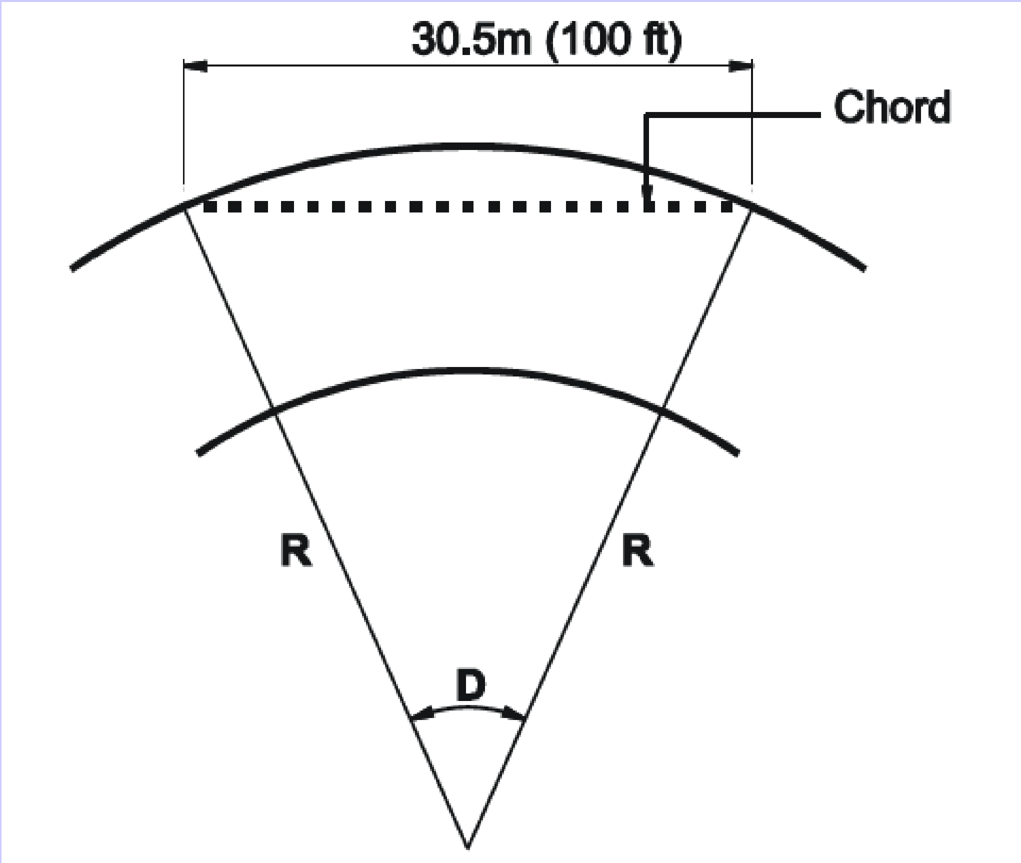
Track Alignment

1- Simple Curve
2- Compound Curve
3- Reverse Curves
4- Superelevation required for such curves
5- Spiral curves as a means of introducing the superelevation on a gradual and uniform basis.
6- Vertical Curves

1- Simple Curve

- A simple curve has a constant radius throughout.

- The degree of curvature generally is measured by the central angle subtended by a 100-ft-long chord



$$2\pi R \rightarrow 360^0,$$

$$\therefore \frac{2\pi R}{360} \rightarrow 1^0 \; and \; \frac{2\pi RD}{360} \rightarrow D^0$$

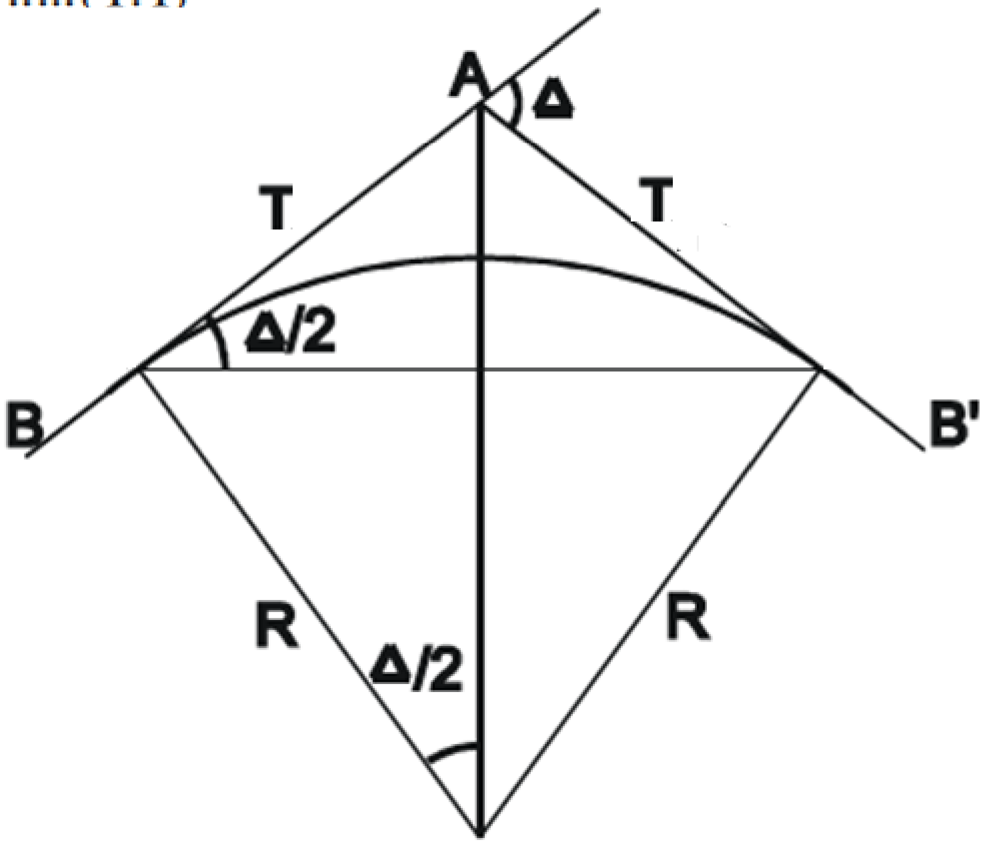
Since arc length of $30.5m$ *corresponds to* D^0,

i.e. $\frac{2\pi RD}{360} = 30.5; \frac{2\pi RD}{360} = D^0$ *solving the equation we get*

$$D = \frac{1750}{R} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.1)$$

$$T = R\tan\frac{\Delta}{2}$$

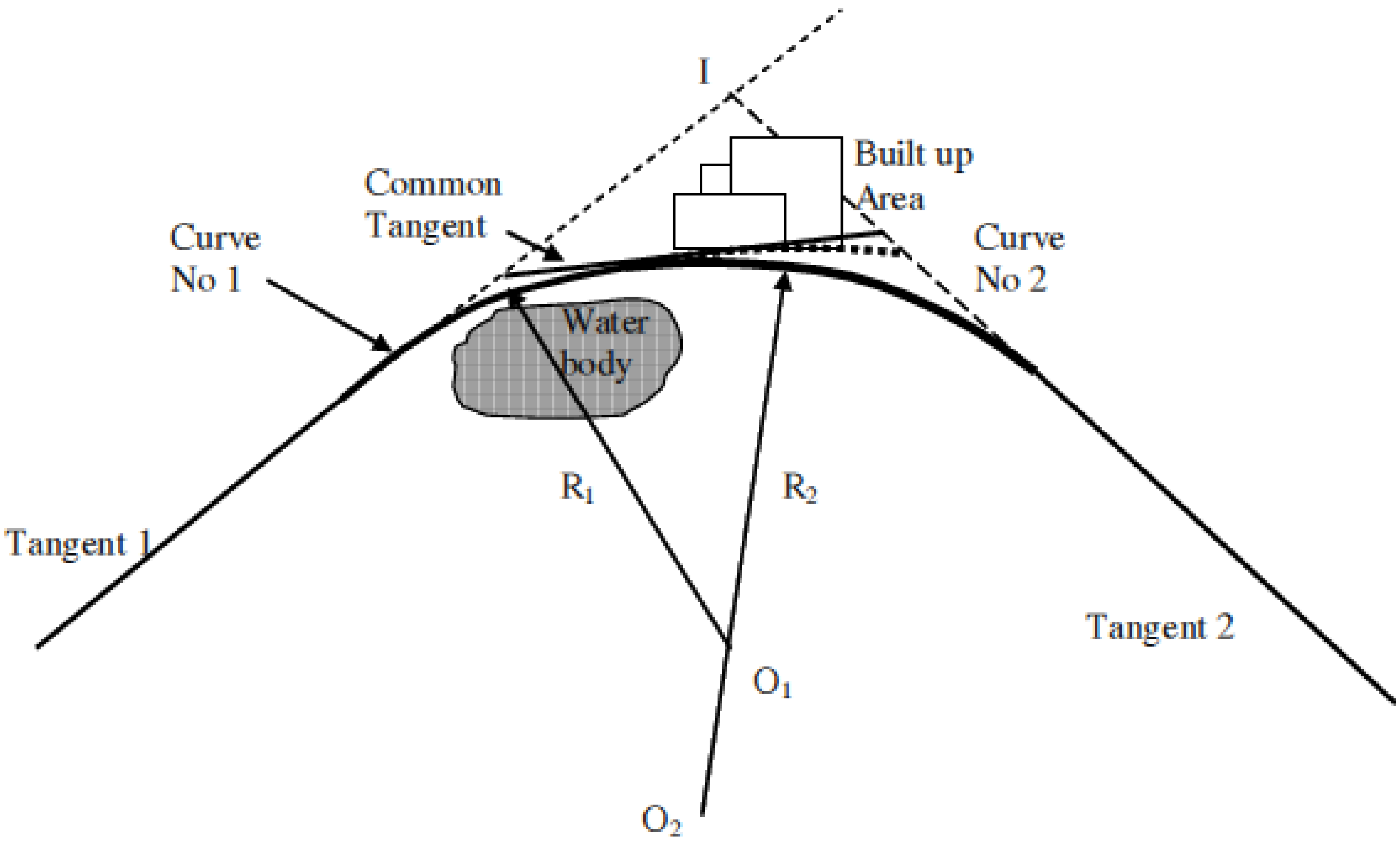
$$L = \frac{100\Delta}{D}$$



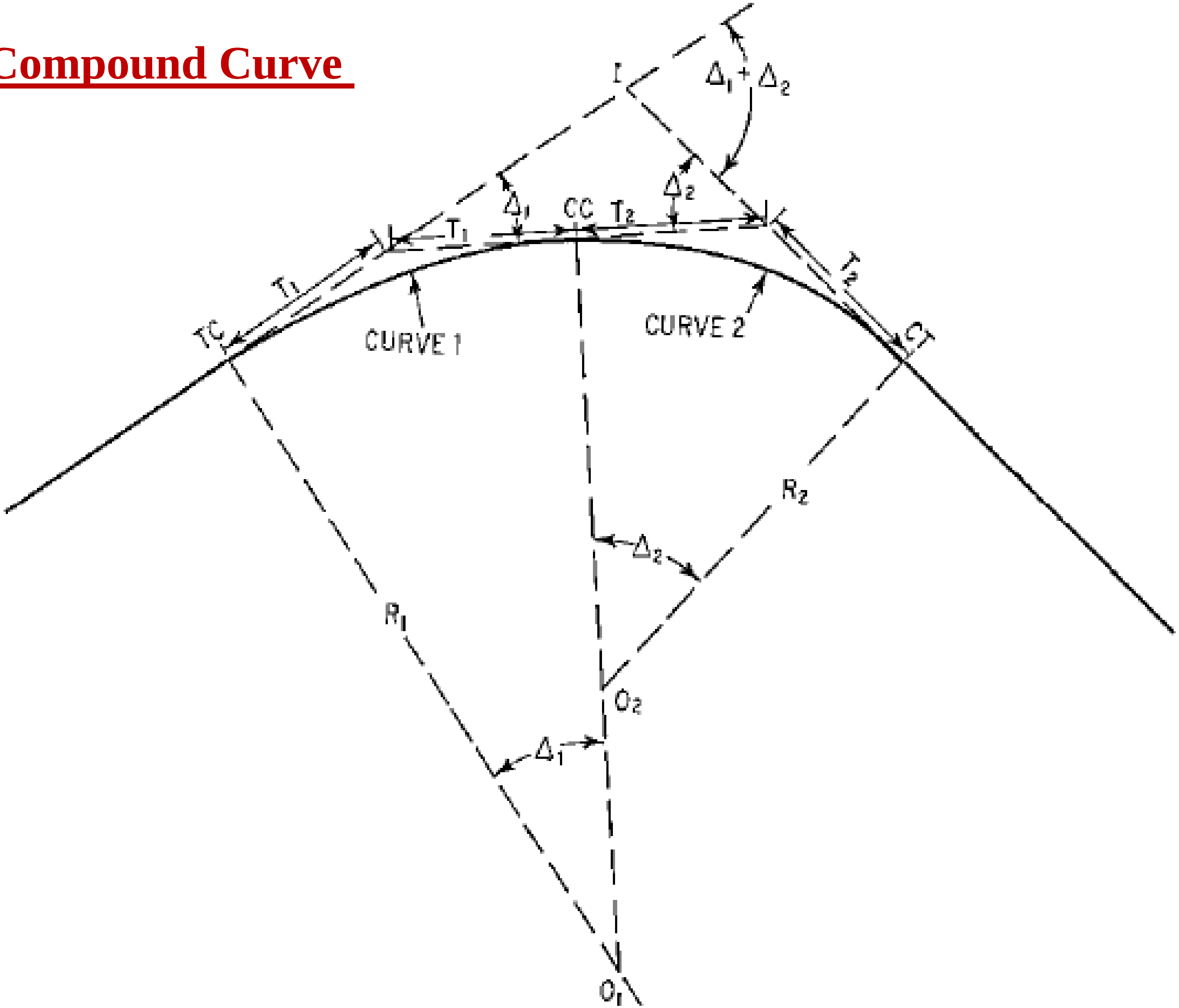
Compound Curve & Reverse Curves

A compound curve comprises two or more simple curves.
Each successive curve having a common tangent with the preceding curve.

Compound curves should be avoided, but they may be used where excessive excavation or fixed objects that must be cleared justify or require such a curve


I
Built up Area
Common Tangent
Curve No 1
Curve No 2
Water body
R1
R2
Tangent 1
Tangent 2
O1
O2

Compound Curve

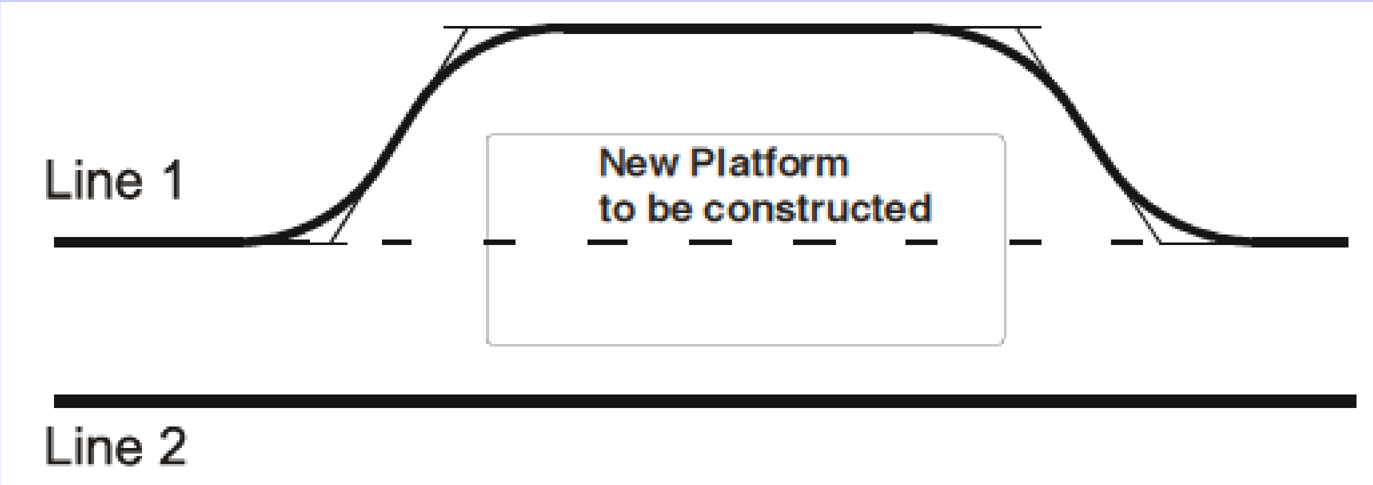


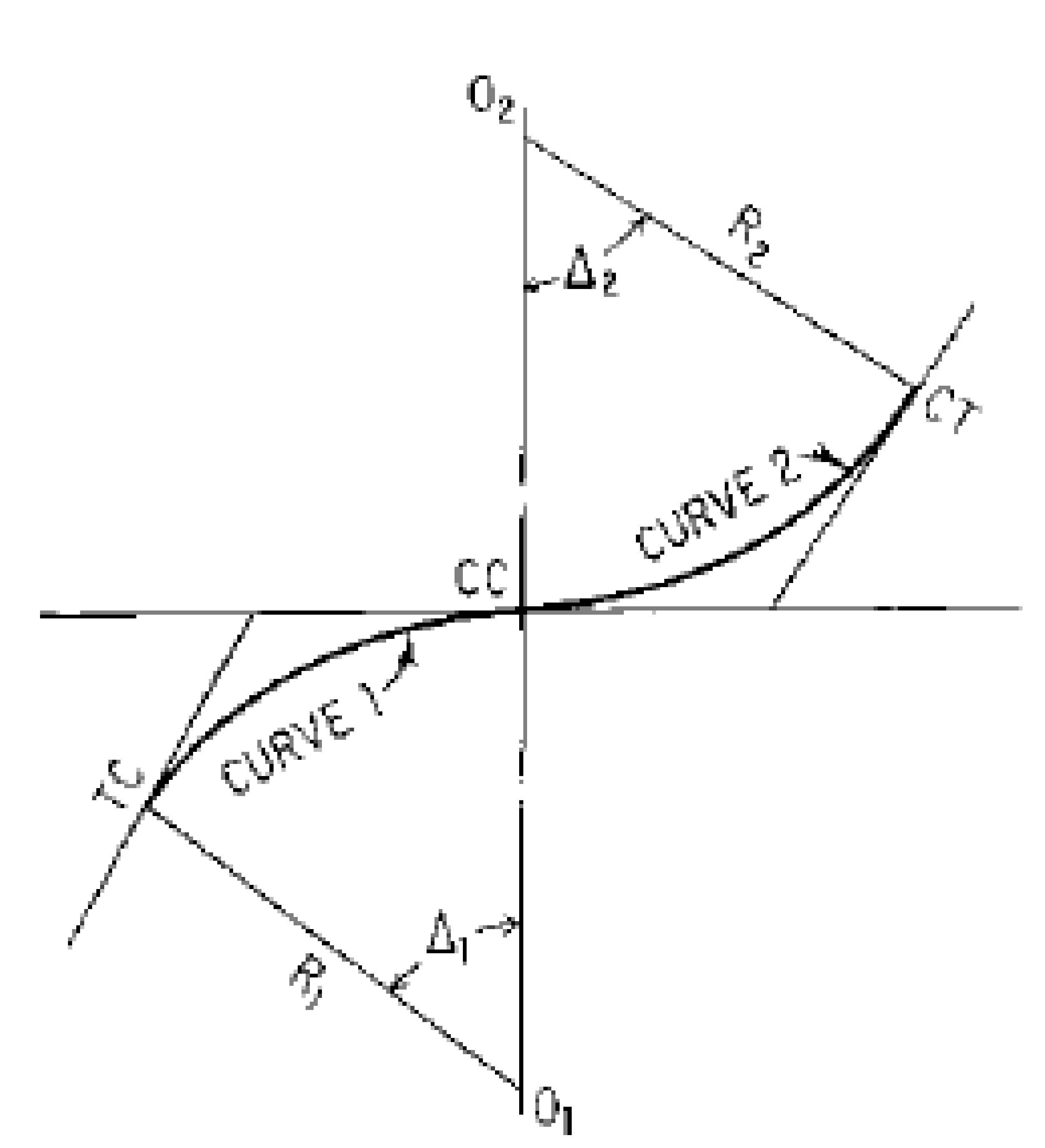
Reverse curve

A reverse curve is a combination of two simple curves with centers on opposite sides of a common tangent

Reverse curves are acceptable in slow-speed passing and yard tracks but should never be used in main line.

A short tangent, at least 100 ft long, but preferably more, should be placed between curves of opposite direction in main line



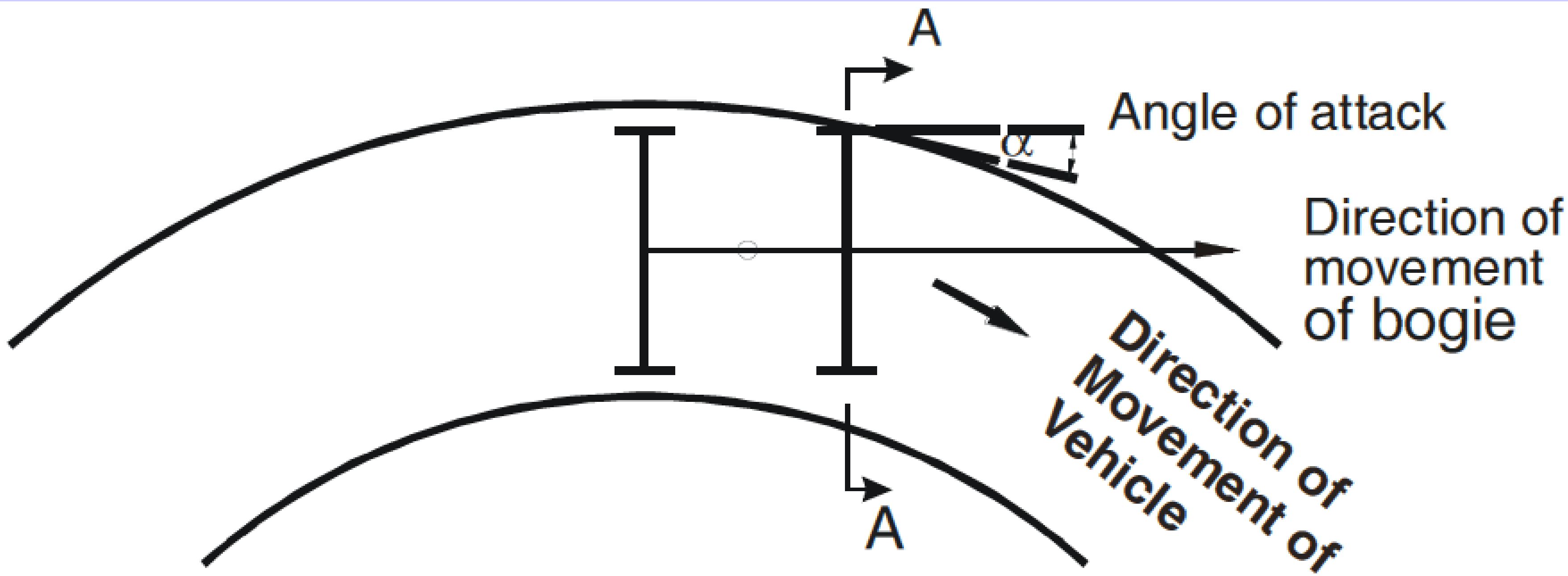


O2
R2
Δ2
CT
CURVE 2
CC
CURVE 1
TC
Δ1
R1
O1

Movement of Vehicle on Curved Track: When a vehicle moves over the curved track, following are to be achieved:

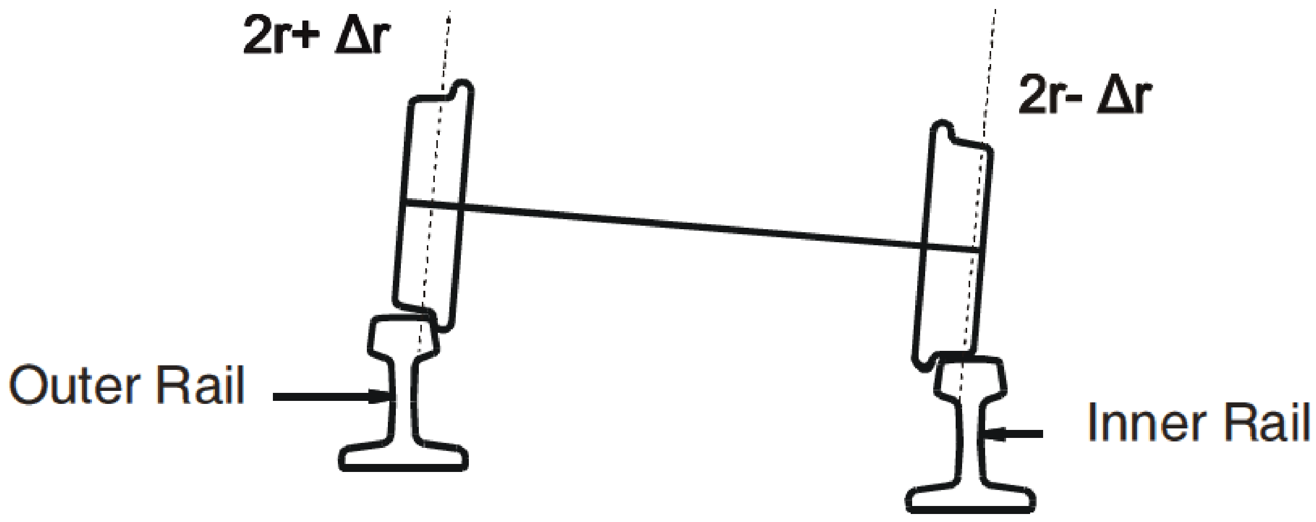
Continuous change in direction

The change in direction of outer rail causes the wheel to change direction. Due to this, there are large lateral forces on the track as well as vehicle.



Movement without slip: On a curved track the length of outer rail is more than the length of inner rail, therefore the outer wheel must travel with a larger radius as compared to the inner wheel.

The actual vehicle movement on curve is quite complicated but we can consider that the vehicle traverses the curved path without appreciable slip due to coning in the wheels, which causes larger diameter to travel on the outer rail and smaller diameter on the inner rail by slight shifting of the center of gravity of the vehicle towards the outer rail.



Where r is radius of wheel at center

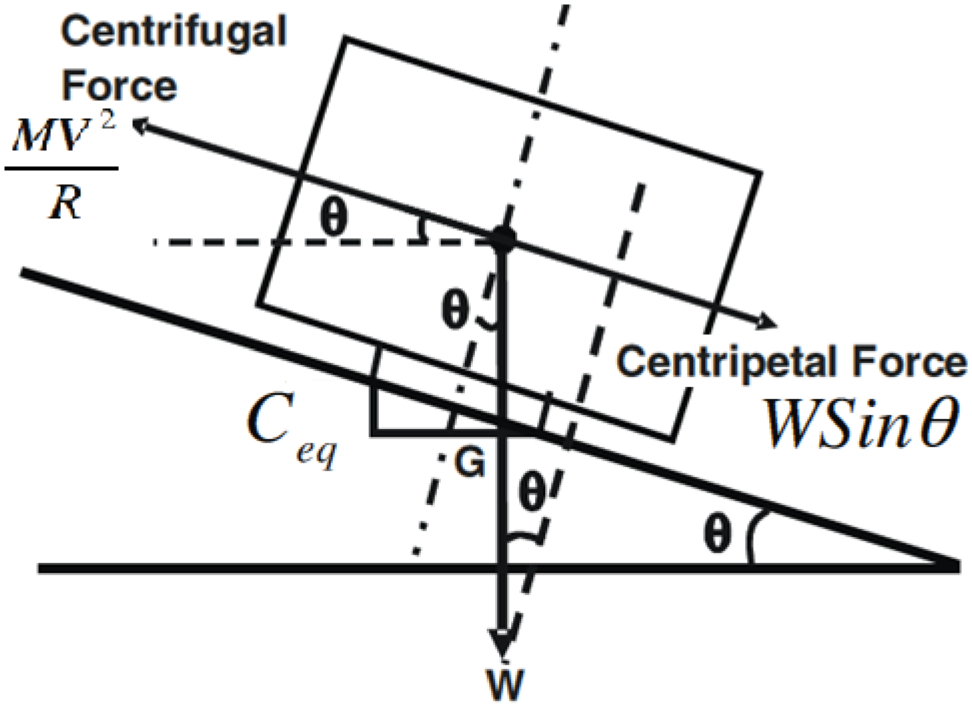
Forces on a Vehicle During Movement on Curve:

The vehicle passing over the curve continuously changes its direction over a curve. Due to inertia, the vehicle tends to continue moving in the straight line but the forced change in direction of the movement by track gives rise to lateral acceleration acting outwards which is felt by the vehicle and all passengers/ things inside. This acceleration is called **Centrifugal Acceleration** and the force due to the same is called **Centrifugal Force**.

$$\frac{MV^2}{R} = WSin\theta$$

$$\Rightarrow \frac{MV^2}{R} = (M * g)\frac{C_{eq}}{G}$$

$$\therefore C_{eq} = \frac{GV^2}{gR}$$



Computation of equilibrium speed: If we provide the equilibrium cant corresponding to the speed of movement of trains, the trains will be moving on the curve without any lateral force and following benefits are there:

- Load is equal on both rails.
- The wear on both rails is equal
- Maintenance of track geometry is easier.
- Fittings and fixtures are subjected to less stresses and wear and tear.
- The passengers are not discomforted.

Trains which are not moving at the equilibrium speed experience either of the two conditions:

a) **Speed of Vehicle More than Equilibrium Speed:** For a vehicle moving at a higher speed than equilibrium speed, the cant required is more than actual cant provided and the difference between the two is called **cant deficiency, C_d**. Such a vehicle experiences outwards lateral force as the centrifugal force is more than centripetal force and in this situation, the wheel load on the outer rail is more than the wheel load on inner rail.

b) Speed of Vehicle Less than Equilibrium Speed Cant Excess: Cant required for a vehicle moving at a lesser speed than equilibrium speed is less than actual cant provided. This difference between the cant actually provided and the cant required for the actual speed is called **cant excess, C_{ex}**. Such a vehicle experiences inwards lateral force as the centrifugal force is less than centripetal force and in this situation, the wheel load on the inner rail is more than the wheel load on outer rail.

$$Equilibium\ \ Cant, C_{eq} = \frac{GV_{eq}{}^{2}}{gR}$$

$$and\ \ Actual\ \ Cant, C_{a} = \frac{GV^{2}}{gR}$$

Where V_{eq} *is Equilibrium speed and*

V *is actual speed of the train*

Subtracting the above two equations,

$$C_{a} - C_{eq} = \frac{GV^{2}}{gR} - \frac{GV_{eq}{}^{2}}{gR}. \ \ Since\ \ C_{d} = C_{a} - C_{eq},$$

$$C_{d} = \frac{G}{g}\left(\frac{V^{2} - V_{eq}{}^{2}}{R}\right)$$

$$C_{d} = \frac{G}{g}(\Delta_{p}),\ \ where\ \ \Delta_{p}\ \ is\ \ the\ \ unbalanced$$

lateral acceleration in $m/\sec^{2}$

1 $i.e.\ \ \Delta_{p} = \frac{C_{d}}{G}g$

Calculating Speed on a Curve: If C_a is the cant actually provided in the track, for a vehicle traveling on the curve at a speed higher than equilibrium speed, the cant deficiency C_d is given by

$$C_d = \frac{GV^2}{gR} - C_a$$

$$\therefore \frac{GV^2}{gR} = C_a + C_d$$

$$\Rightarrow V = \sqrt{\frac{g}{G}} * \sqrt{(C_a + C_d) * R}$$

Substituting g = 9.81 m/sec^2, G = 1500 mm and adopting C_a in mm, V in KMPH, and R in m, and accounting for the units, the equation reduces to

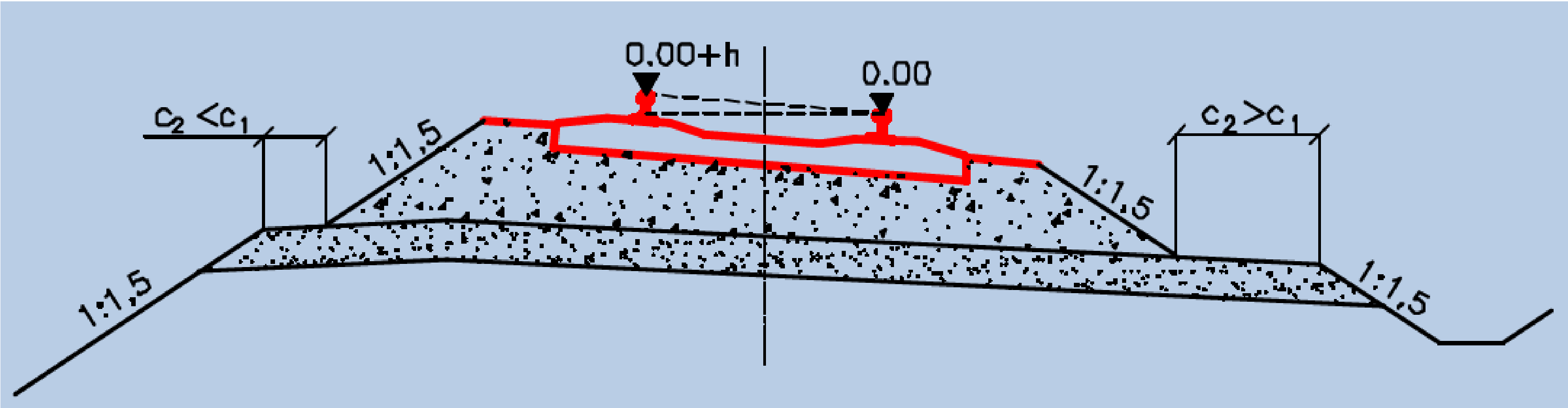
$$V = 0.29 * \sqrt{(C_a + C_d) * R}$$

Remarks

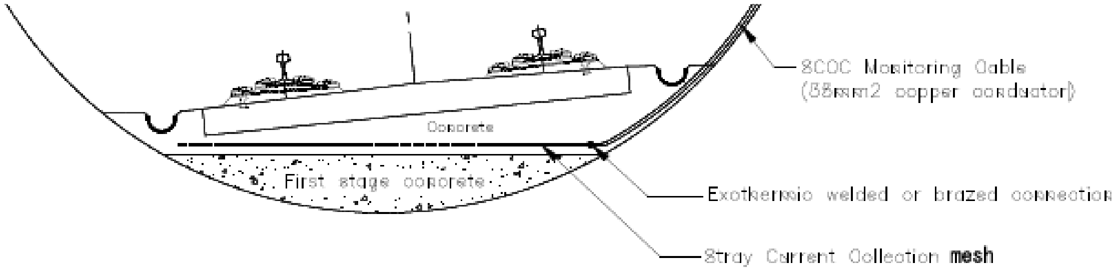
1. The cant is NOT applied:
   - in curves of side tracks on stations (low speed),
   - in curves of turnouts if the main track is straight,
   - in curves if the speed is lower than 30 km/h,
   - on sidings shorter than 1 km.
2. If the maximum calculated cant < 20 mm then h=0 mm is applied.
3. If the minimum calculated cant > 150 mm then h=150 mm is applied and the maximum speed should be delimited.
4. At the end of calculation it should be checked if the eccentric accelerations are smaller than allowable ones:

$$a_p = \frac{v_{max}^2}{12.96\,R} - \frac{C}{153} \le a_{dop} \qquad a_t = \frac{C}{153} - \frac{v_t^2}{12.96\,R} \le a_t$$

How to aply the cant?



1. The designed cant should fall into range from 20 to 150 mm -> safety.
2. The cant amount should be rounded to 5 mm -> technical limitations.
3. Analysis of the cant should take into account the true traffic structure and the real speeds of the passenger and freight trains.
4. The applied cant should allow shape the slopes of the ballast to save at least 60 cm of the cess on the trackway.



** قيم العجلة الجانبية المؤثرة على راحة الركاب

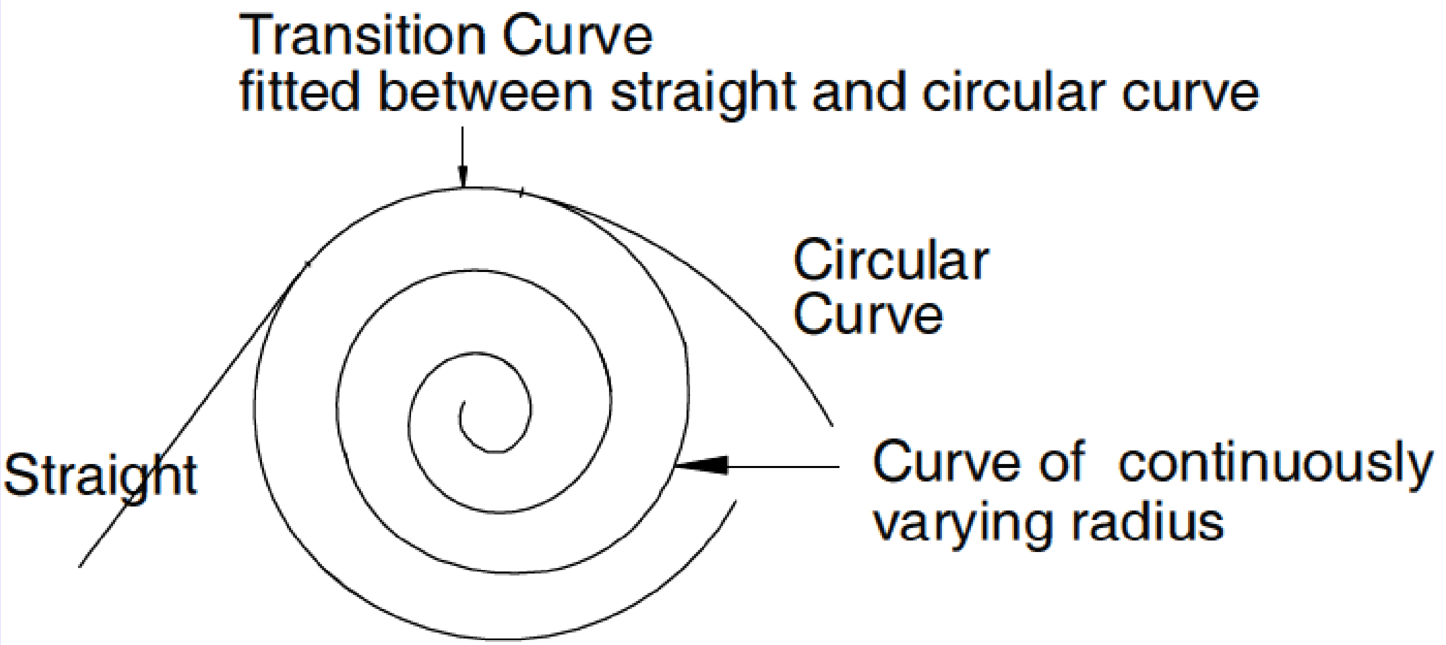
تعتمد راحة الراكب علي قيمة العجلة الجانبية وفترة تأثيرها والتردد المحتمل لجسم الانسان واتجاه العجلة. فقد وجد أن عجلة تساوي ١ م/ث ٢ عند ١.٥ هرتز يمكن تحملها لفترة ٥ ساعات ونصف في الاتجاه الرأسي، ولفترة ٣ ساعات ونصف في الاتجاه الأفقي. والحد الأقصي المسموح به للعجلة الجانبية المؤثرة علي الراكب هو ١ م/ث ٢

ولراحة الراكب فإن العجلة الجانبية المتولدة يجب ألا تتعدي ثلثي العجلة التي يتحملها الراكب.

وتعطي القيمة الدنيا لإرتفاع الظهر عن البطن بالمعادلة:

$$C_{min} = \frac{11.8\, V^2}{R} - 153\, \alpha$$

منحنى الانتقالTransition curves



The equation of this curve is:

$$l \alpha \frac{1}{r} \quad \text{i.e. } l\,r = \text{constant}$$

where l is the distance traveled on the transition curve and r is the radius at the distance l from the start of the transition curve. The curve described by the eq. is called a clothoid or Euler spiral.

Parameters for design of a transition curve:

R_{ca} : **Rate of change of actual cant over transition**

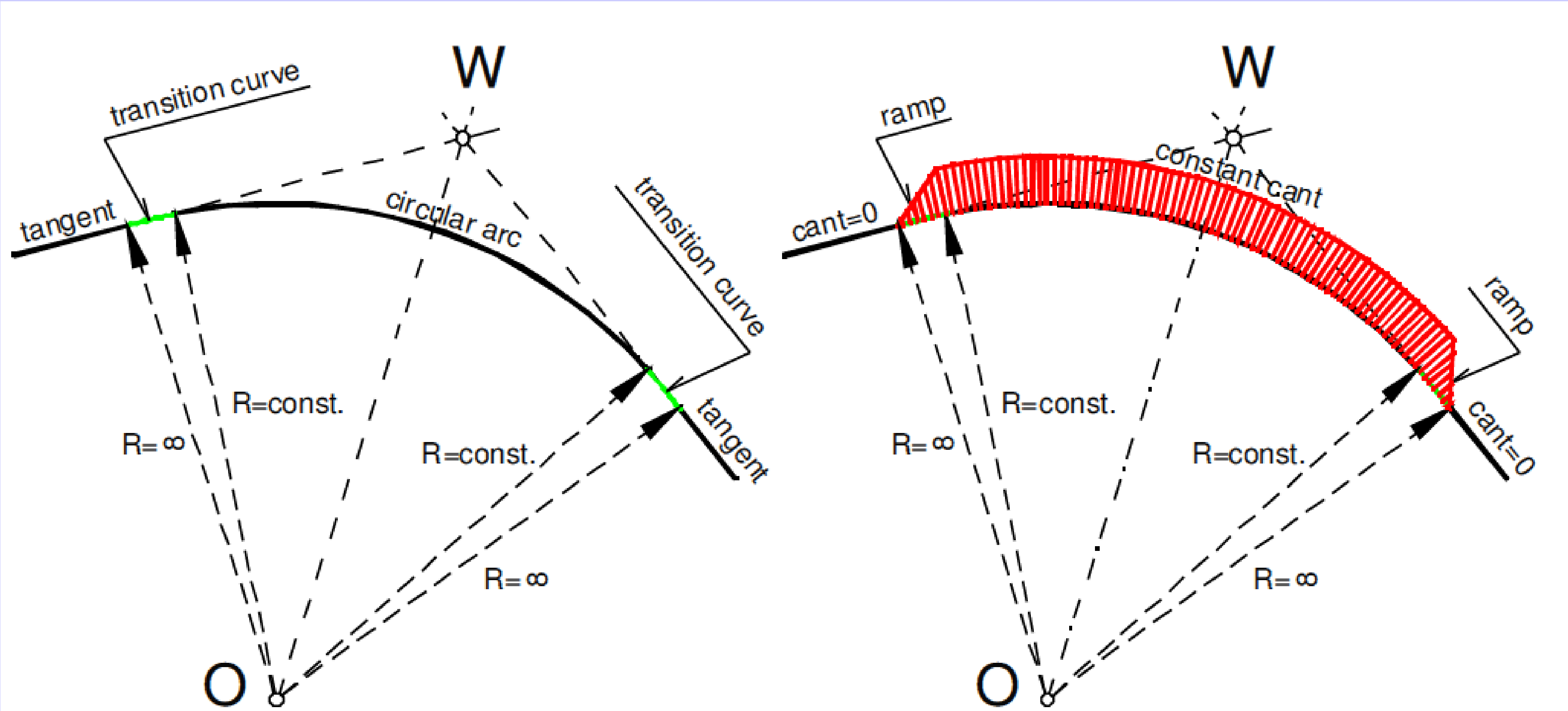
R_{cd} : **Rate of change of cant deficiently over transition**

i : **Cant gradient** i.e. rate of change of actual cant over length of transition.

Desirable features of a transition are:

a) In order that there is no jerk at the either end of the transition, the transition curve must be tangential to the straight as well as the circular curve.

b) It shall ensure a gradual increase of curvature (= 1/Radius) from zero at tangent point (Radius = ∞) to the specified curvature 1/R for the circular curve.


W
transition curve
tangent
circular arc
transition curve
R=const.
R= ∞
R=const.
tangent
R= ∞
O
W
ramp
cant=0
constant cant
ramp
R=const.
R= ∞
R=const.
cant=0
R= ∞
O

$$C_d = \frac{G}{g}(\Delta p)$$

At start of transition, the vehicle is on the straight,

$\therefore$ Cant Deficiency = 0.

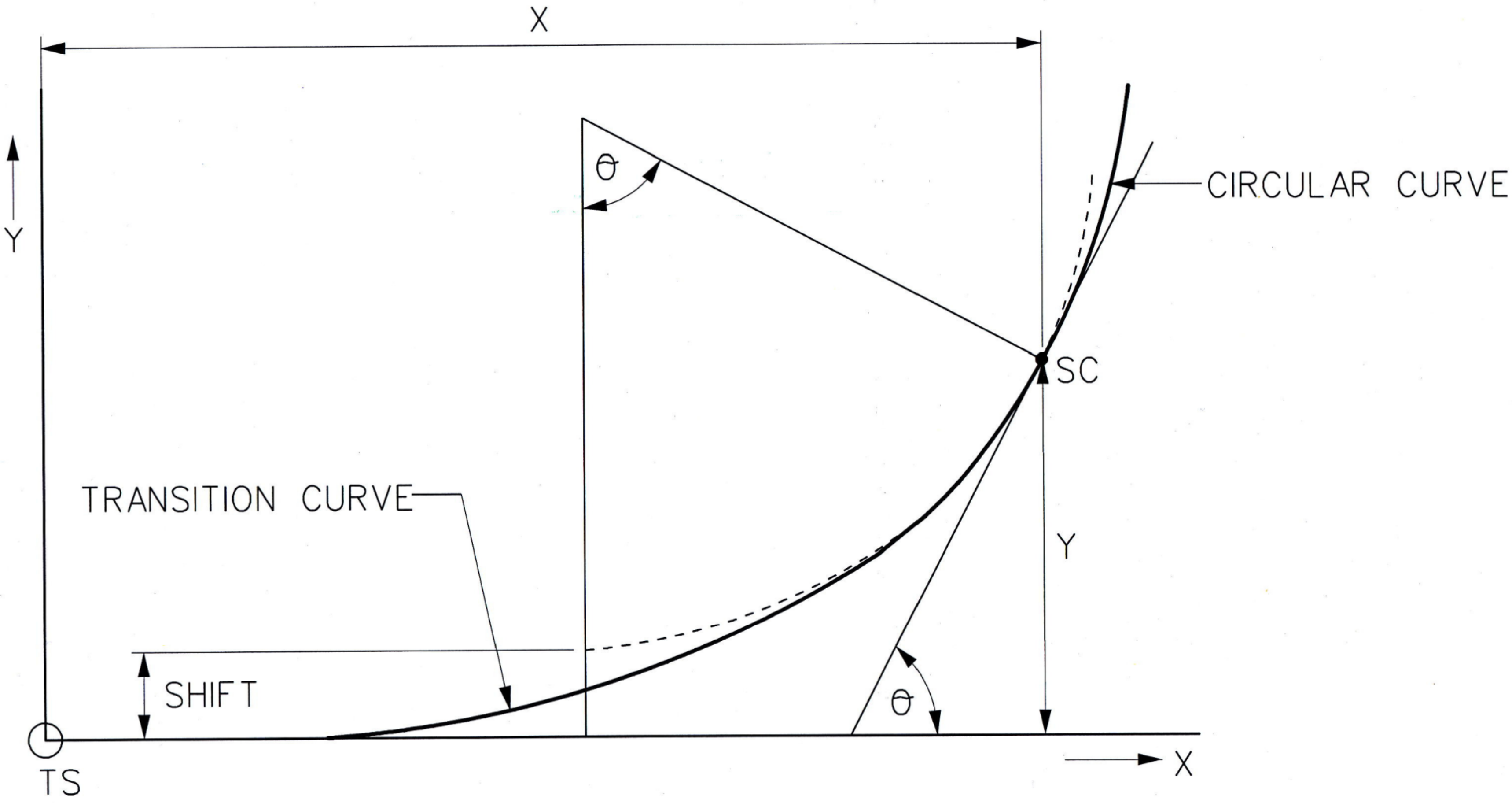
For a vehicle traveling at speed V, time of travel on the transition curve of length L is L/V.

$\therefore$ The rate of change of cant deficiency over the transition:

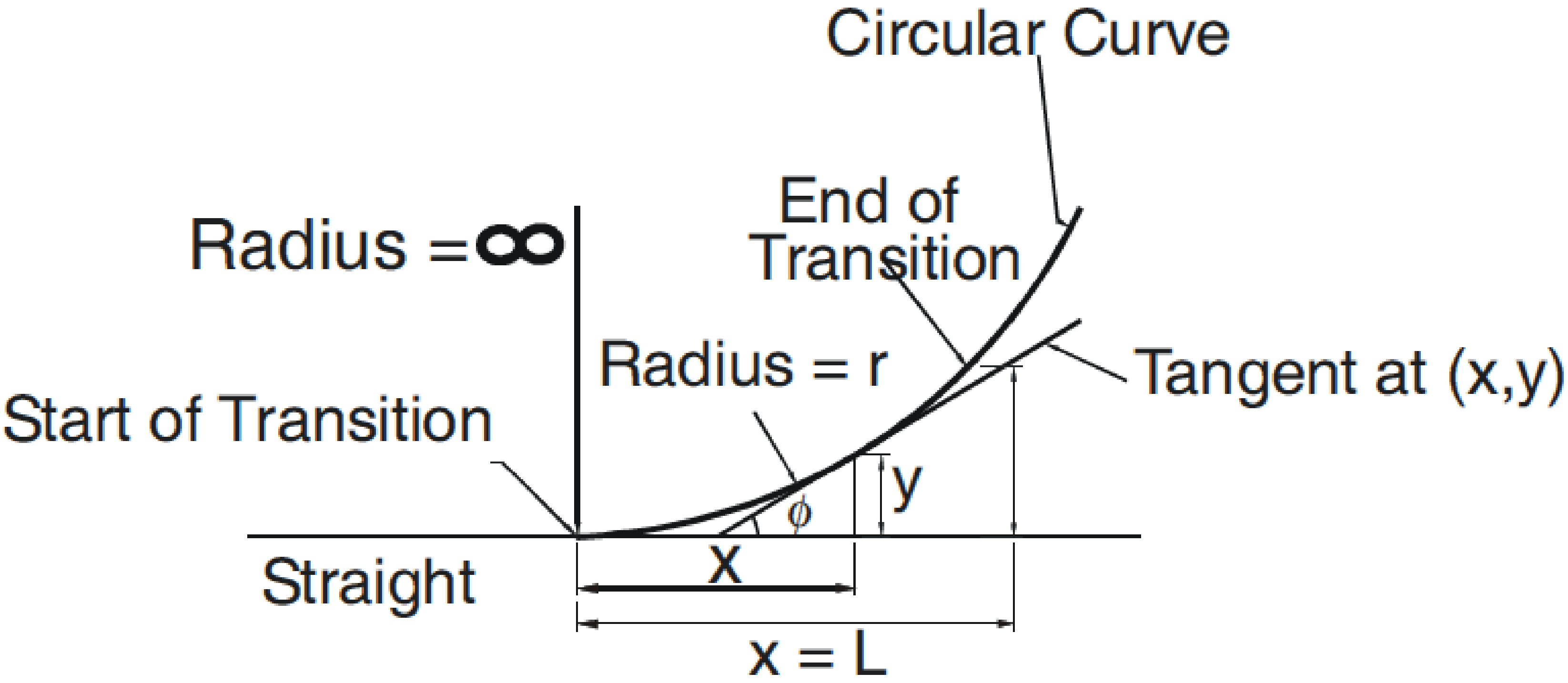
$$\frac{C_d - 0}{L/V} = \frac{G}{g}\left[\frac{\Delta p}{\left(L/V\right)}\right]$$

$$\text{i.e. } r_{cd} = \frac{G}{g}\left[\text{rate of change of unbalanced lateral acceleration}\right]$$

$$\text{i.e. } r_{cd} = \frac{1500\ \text{mm}}{9.81\,\text{m/sec}^2}\left[0.3\,\text{m/sec}^3\right] \cong 46\ \text{mm/sec}$$


X
θ
CIRCULAR CURVE
Y
SC
TRANSITION CURVE
Y
SHIFT
θ
TS
X

Cubic Parabola as transition:



$x * r$ =Constant.

Now, at x = L, r=R therefore, RL = Constant.

i.e. $x * r = RL$ and $\therefore \dfrac{1}{r} = \dfrac{x}{RL}$

If ϕ is the angle subtended by the tangent at any point on curve (see figure 1.13), the curvature is given by $\frac{1}{r} = \frac{d\phi}{dx}$

Integrating, we get $\phi = \frac{x^2}{2RL}$ + Constant1

At x = 0, transition is tangential to the tangent track, so $\phi = 0$

∴ Constant1 = 0 in above equation

i.e. $\phi = \frac{x^2}{2RL}$

Now, $\phi = \frac{dy}{dx}$ $\therefore$ $\phi = \frac{dy}{dx} = \frac{x^2}{2RL}$.

On integrating, y = $\frac{x^3}{6RL}$ + Constant2

At x = 0, y is 0; so the Constant2 in above equation becomes 0

$\therefore$ The equation reduces to y = $\frac{x^3}{6RL}$

Finding Length of transition:

Desirable Length based on criterion of rate of change of cant deficiency:

If we consider the length of transition and the speed of travel, the time of travel of a vehicle on the transition curve is L/V.

And from the rate of change of cant deficiency criteria, time of travel on the curve is C_d/r_{cd}.

Equating, $\dfrac{C_d}{r_{cd}} = \dfrac{L}{V}$, $\therefore L = C_d * V \ / r_{cd}$

Taking value of r_{cd} as 35 mm/sec, and taking care of the units,

$L = 0.008\ C_d \times V_m$

where C_d is in mm and V_m is maximum speed on curve section in kmph and L is in metres.

Desirable Length based on Criterion of rate of change of actual cant:

Since the value of r_{ca} and r_{cd} are similar,

$L = 0.008\ C_a \times V_m$

where C_a is in mm and V_m is maximum speed on curve section in kmph and L is in metres

Length based on Criterion of cant gradient

If i is the cant gradient permissible, the length of the transition shall be

$L = i * C_a$.

Desirable value of i is 1 in 720. Substituting the same and taking care of the units, $L= 0.72* C_a$

where C_a is in mm and L is in metres.

Length of transition in case of compound and reverse curves.

the desirable length of transition, L in m, between the compound curves shall be more than the maximum of following three values

a) $L = 0.008 \times (C_{a1} - C_{a2}) \times V_m$

b) $L = 0.008 \times (C_{a1} - C_{d2}) \times V_m$

c) $L = 0.72 \times (C_{a1} - C_{a2})$

Desirable length of transition between reverse curves

If there is no straight between reverse curves, the two curves joining at the transition have curvature in different directions. Therefore, the desirable length of transition, L, between the reverse curves shall be more than the maximum of the following three values :

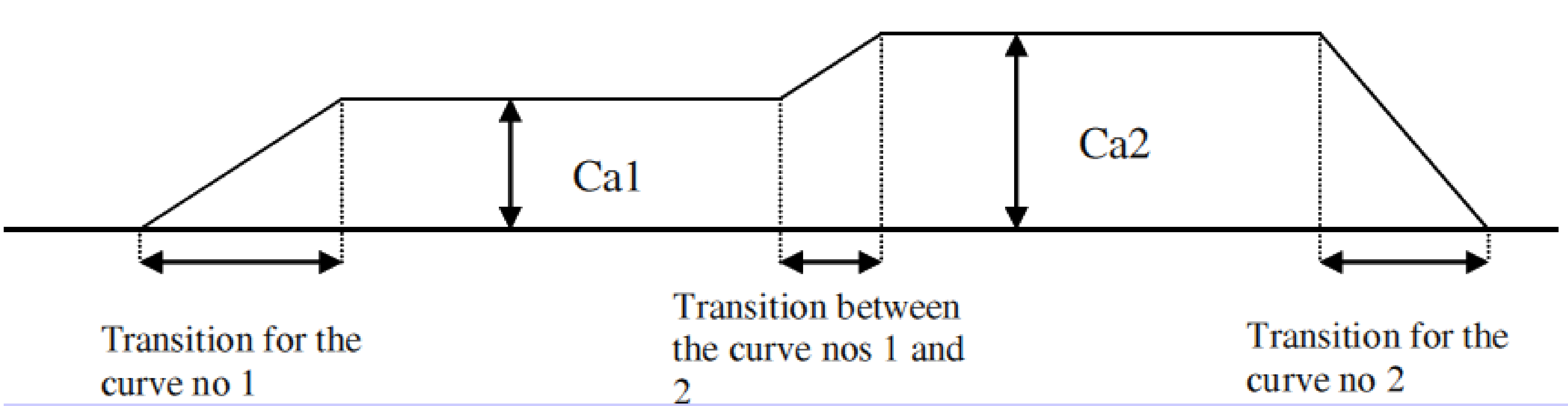
a) $L = 0.008 \times (C_{a1} + C_{a2}) \times V_m$

b) $L = 0.008 \times (C_{d1} + C_{d2}) \times V_m$

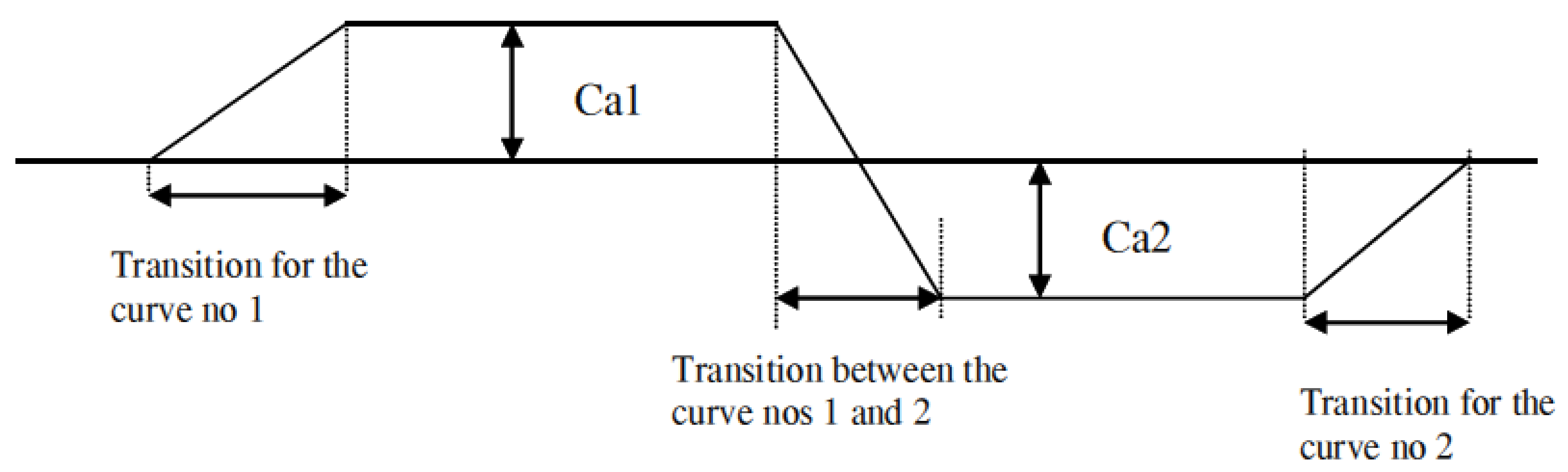
c) $L = 0.72 \times (C_{a1} + C_{a2})$

Where C_{a1} and C_{a2} refer to the actual cant in mm for curve no 1 and curve no 2, C_{d1} and C_{d2} refer to cant deficiency in mm for curve no 1 and 2 which form the compound/ reverse curve.
and V_m = Maximum permissible speed on the curve (KMPH)

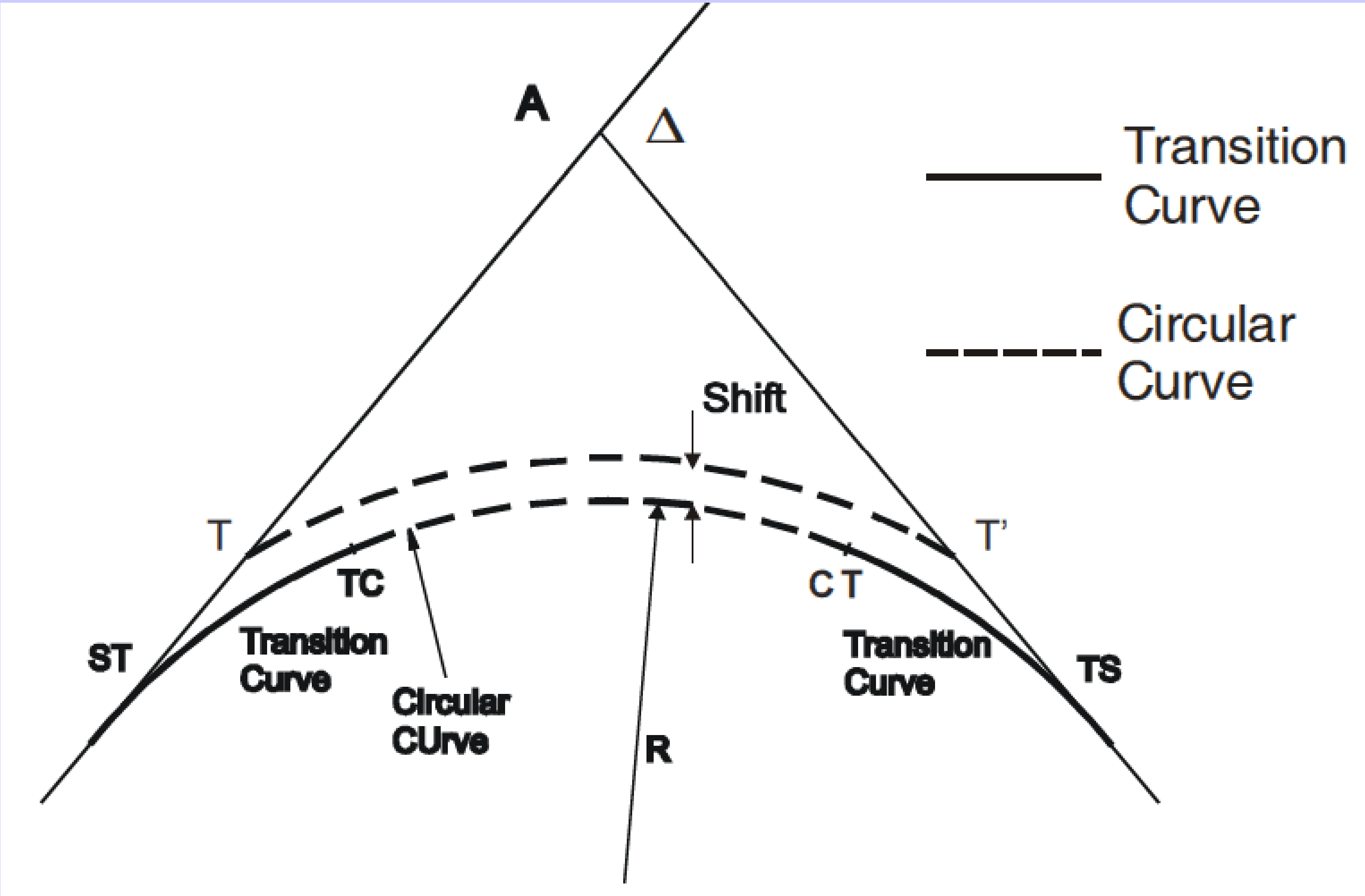
Curvature of Compound curve

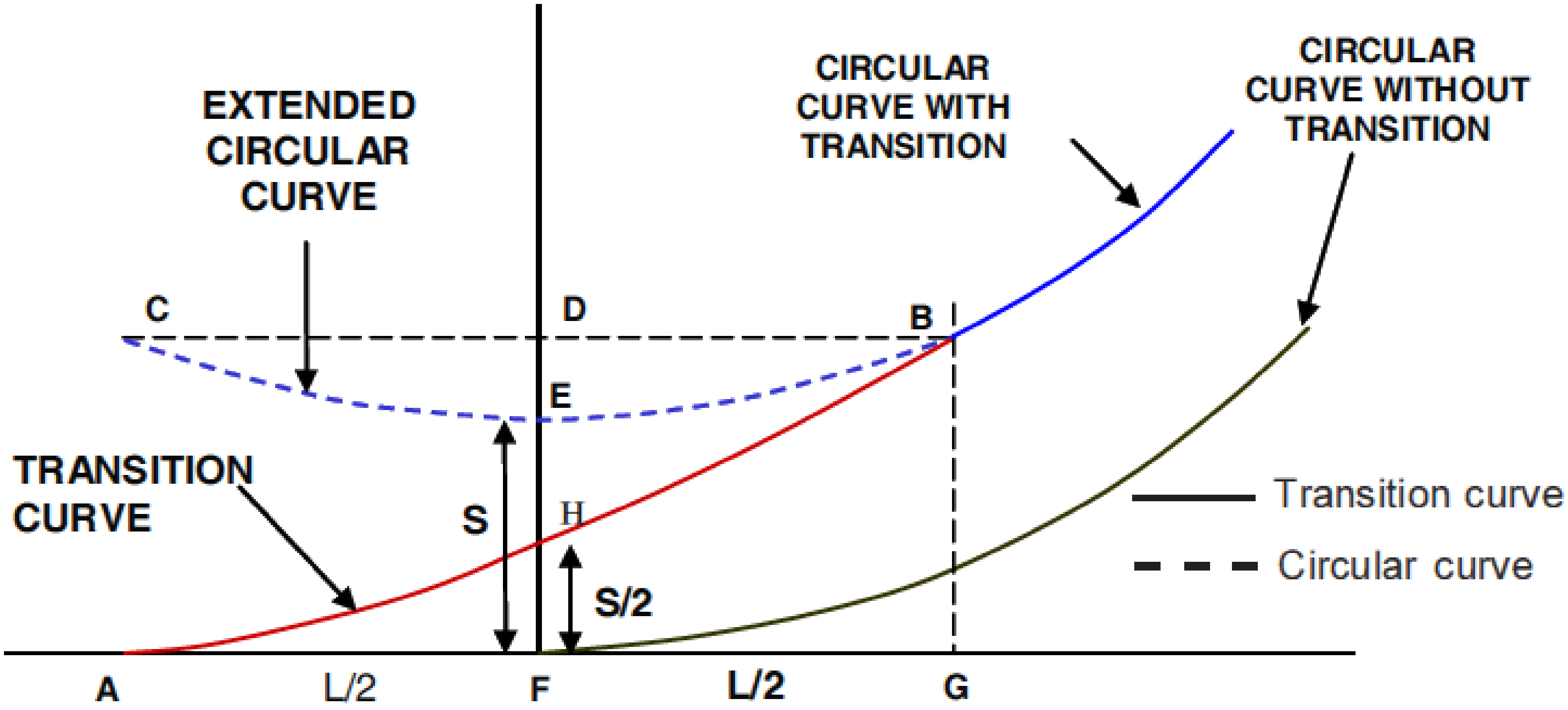


Curvature of Reverse curve



The Shift


A
Δ
Transition Curve
Circular Curve
Shift
T
T'
TC
C T
ST
Transition Curve
Circular CUrve
R
Transition Curve
TS



shift is measured between the original circular curve without transition and the circular curve after the transition has been provided, and not between the straight and the circular curve i.e. shift is EF and not BG.)

Now since EF = DF – DE = BG – DE

for the transition, $y = \frac{x^3}{6RL}$

$BG = y_{(x=L)} = \frac{L^3}{6RL} = \frac{L^2}{6R}$

For the arc BEC of the circular curve, BC is the chord with Length equal to the length of transition, L,

$\therefore$ DE is the versine for the chord i.e. DE= $\frac{L^2}{8R}$

$\therefore$ EF=BG-DE = $\frac{L^2}{6R} - \frac{L^2}{8R}$

i.e Shift = $\frac{L^2}{24R}$

Considering $FH = Y_{(x=L/2)} = \frac{(L/2)^3}{6RL} = \frac{L^2}{48R}$

i.e. The offset in the transition curve at station 0 from straight is equal to half the shift of circular curve.

The value of shift depends on:

i) Radius, R

ii) Length of transition, L

Vertical curves

The radius of the vertical curve can be worked out based on the following relationship between speed of the vehicles, radius of the vertical curve and permissible values of vertical accelerations. $R_v \geq V^2_m / a_m$

Where, R_v = Radius of vertical curve in meter لايقل عن ٢٠٠٠ متر.

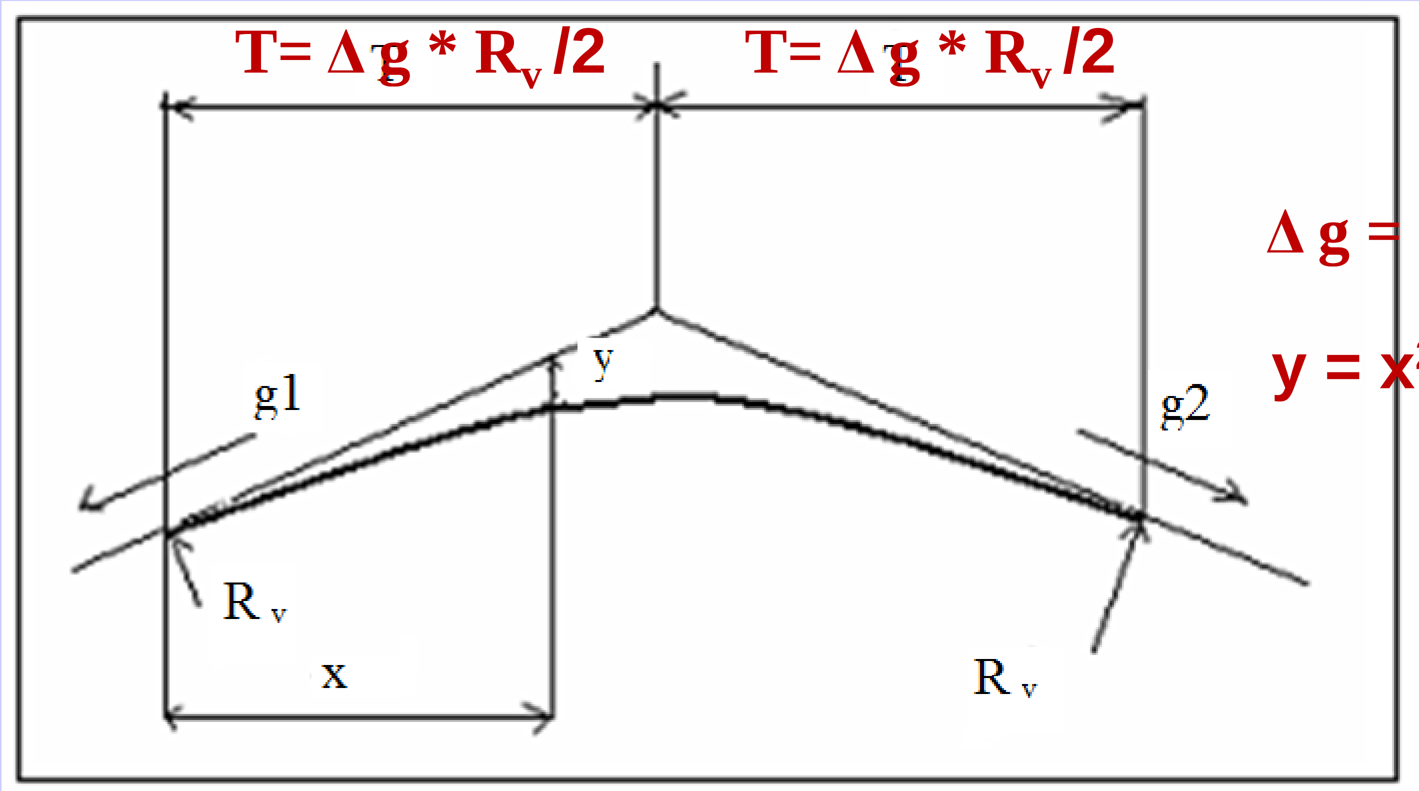
V_m = maximum permissible speed of the vehicle in m/sec

a_m = permissible vertical acceleration in m/sec^2

The limits of vertical acceleration are

generally accepted as 0.3 to 0.45 m/sec^2.

The vertical curves are to be provided only when the algebraic difference in gradients meeting at a point is equal to or more than 4 mm per metre or 0.4 percent .



$\Delta g = (g2 \pm g1)$

$y = x^2 \; / \; 2 R_v$